Uniform Circular Motion and Torque

Physics

Unit 3

NAD 2023 Standard F4 (Torque)

Credits

- This Slideshow was developed to accompany the textbook
  - *OpenStax High School Physics*
    - *Available for free at https://openstax.org/details/books/physics*
    - *By Paul Peter Urone and Roger Hinrichs*
    - *2020 edition*
- Some examples and diagrams are taken from the *OpenStax College Physics, Physics,* and *Cutnell & Johnson Physics* 6th ed.

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After this lesson you will…

- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity
- Establish the expression for centripetal acceleration.

3-01 ROTATION ANGLE AND CENTRIPETAL ACCELERATION

OpenStax High School Physics 6.1, 6.2
OpenStax College Physics 2e 6.1-6.2

3-01 Rotation Angle and Centripetal Acceleration

- Newton's Laws of motion primarily relate to straight-line motion.
- Uniform Circular Motion
  - Motion in circle with constant speed
- Rotation Angle ($\Delta\theta$)
  - Angle through which an object rotates

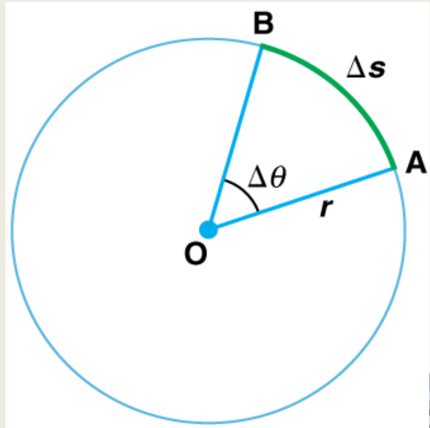
3-01 Rotation Angle and Centripetal Acceleration

- Arc Length is the distance around part of circle

$$\Delta\theta = \frac{\Delta \boldsymbol{s}}{\boldsymbol{r}}$$

- Angle Units:
  - Revolutions: 1 circle = 1 rev
  - Degrees: 1 circle = 360°
  - Radians: 1 circle = 2π
- Arc Length formula must use radians for the angle unit

$$2\pi = 360° = 1\,rev$$



3-01 Rotation Angle and Centripetal Acceleration

- Convert 60° to radians
- Convert 2 revolutions to radians

$$\frac{60°}{}\left(\frac{2\pi}{360°}\right) = \frac{\pi}{3}$$

$$\frac{2\,rev}{}\left(\frac{2\pi}{1\,rev}\right) = 4\pi$$

3-01 Rotation Angle and Centripetal Acceleration

- Angular Velocity (ω)
  - How fast an object rotates

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Unit: rad/s
  - CCW +, CW –

$$v = \frac{\Delta s}{\Delta t}$$

$$\Delta\theta = \frac{\Delta s}{r} \rightarrow \Delta s = r\Delta\theta$$

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega$$

3-01 Rotation Angle and Centripetal Acceleration

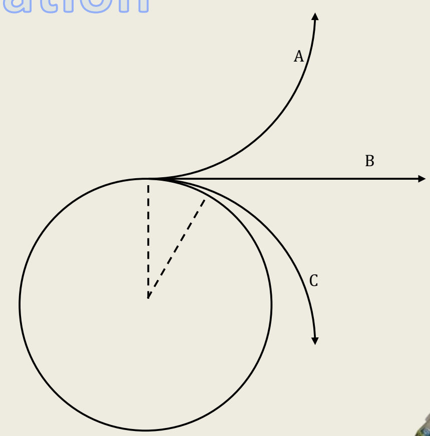
- A CD rotates 320 times in 2.4 s. What is its angular velocity in rad/s? What is the linear velocity of a point 5 cm from the center?

$$\theta = 320\, rev\, (2\pi/1\, rev) = 640\pi\, rad$$
$$t = 2.4s$$
$$\omega = \theta/t = 640\pi\, rad/2.4s = 838\, rad/s$$

$$v = r\omega$$
$$v = (0.05\, m)\left(838 \frac{rad}{s}\right) = 41.9\, m/s$$

3-01 Rotation Angle and Centripetal Acceleration

- Make a hypothesis about what will happen. Which path will an object most closely follow when the centripetal force is removed?



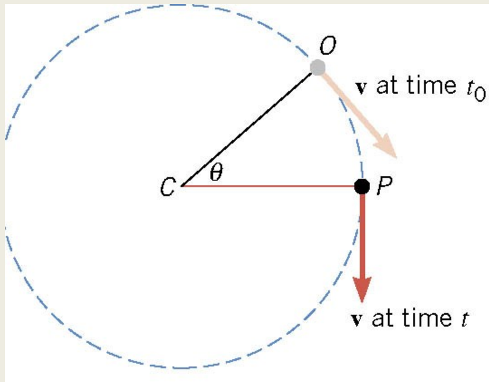
3-01 Rotation Angle and Centripetal Acceleration

1. Put the plate on a flat surface and put a marble in the ridge.
2. Push the marble in the ridge so that it travels around the plate and then out of the removed section.
3. What is providing the centripetal force? i.e. what is keeping the marble traveling in a circle?
4. Perform the test several times and record your results.
5. Which of Newton's Laws explains the results?
6. This would have been more complicated if the object moved in a vertical circle. Why?

3. Rim of the plate
4. Straight line (B)
5. 1st
6. Gravity would have pulled it down

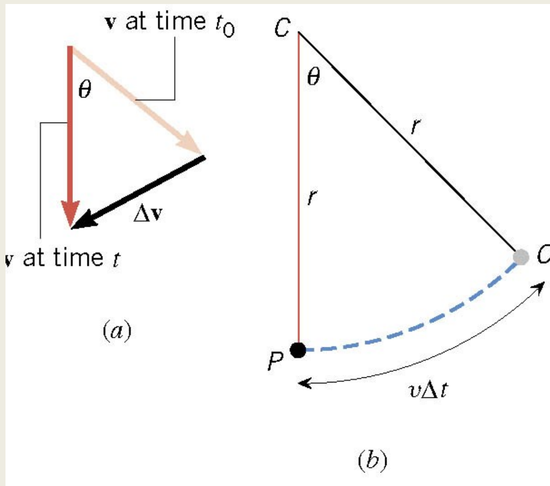
3-01 Rotation Angle and Centripetal Acceleration

- Object moves in circular path
- At time t_0 it is at point O with a velocity tangent to the circle
- At time t, it is at point P with a velocity tangent to the circle
- The radius has moved through angle θ



3-01 Rotation Angle and Centripetal Acceleration

- Draw the two velocity vectors so that they have the same tails.
- The vector connecting the heads is Δv
- Draw the triangle made by the change in position and you get the triangle in (b)



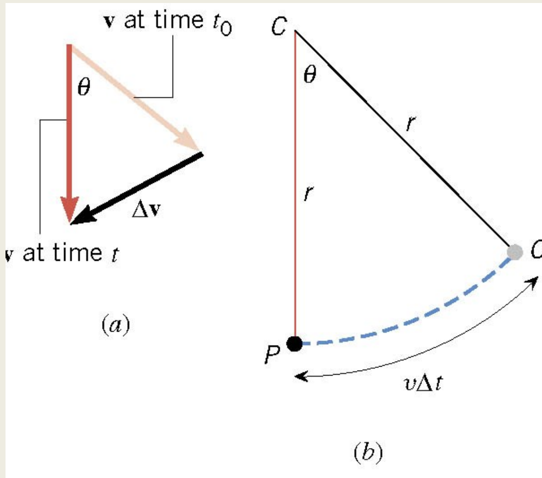
3-01 Rotation Angle and Centripetal Acceleration

- Since the triangles have the same angle are isosceles, they are similar.

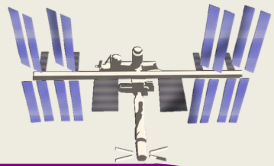
$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

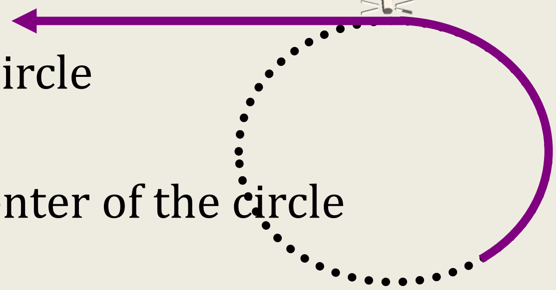
$$a_C = \frac{v^2}{r} = r\omega^2$$



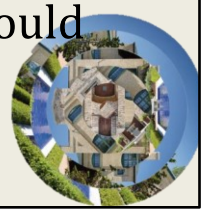
3-01 Rotation Angle and Centripetal Acceleration



- At any given moment
  - $\boldsymbol{v}$ is pointing tangent to the circle
  - $\boldsymbol{a_c}$ is pointing towards the center of the circle



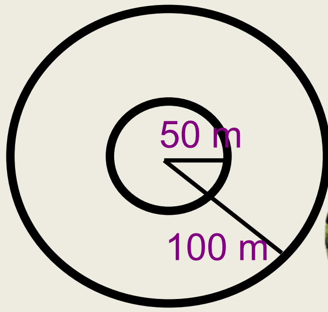
- If the object suddenly broke from circular motion would travel in line tangent to circle



Have a string with something soft on end.
Swing it and let go to illustrate.

3-01 Rotation Angle and Centripetal Acceleration

- Two identical cars are going around two corners at 30 m/s. Each car can handle up to 1 g. The radius of the first curve is 50 m and the radius of the second is 100 m. Do either of the cars make the curve? (hint find the a_c)



$$a_{c1} = \frac{v^2}{r} \rightarrow a_{c1} = \frac{\left(30\ \frac{m}{s}\right)^2}{50\ m} \rightarrow a_{c1} = 18\ \frac{m}{s^2}$$

Can’t make it

$$a_{c2} = \frac{\left(30\ \frac{m}{s}\right)^2}{100\ m} = 9\ \frac{m}{s^2}$$

Yes

F = ma

After this lesson you will…

- Apply centripetal force

3-02 CENTRIPETAL FORCE

OpenStax High School Physics 6.2
OpenStax College Physics 2e 6.3

3-02 Centripetal Force

F = ma

- Newton's 2nd Law
  - Whenever there is acceleration there is a force to cause it

- $F = ma$
- $F_C = ma_C$

$$F_C = \frac{mv^2}{r} = mr\omega^2$$

3-02 Centripetal Force

F = ma

- Centripetal Force is not a new, separate force created by nature!
- Some other force creates centripetal force
  - Swinging something from a string → tension
  - Satellite in orbit → gravity
  - Car going around curve → friction

3-02 Centripetal Force

F = ma

- A 1.25-kg toy airplane is attached to a string and swung in a circle with radius = 0.50 m. What was the centripetal force for a speed of 20 m/s? What provides the F_c?

- F_c = 1000 N
- Tension in the string

$$F_C = \frac{mv^2}{r}$$
$$= \frac{(1.25\ kg)\left(20\frac{m}{s}\right)^2}{0.50\ m}$$
$$= 1000\ N$$

3-02 Centripetal Force

F = ma

- What affects F_c more: a change in mass, a change in radius, or a change in speed?
- A change in speed since it is squared and the others aren't.

3-02 Centripetal Force

F = ma

- Why do objects seem to fly away from circular motion?

- They really go in a straight line according to Newton's First Law.

3-02 Centripetal Force

F = ma

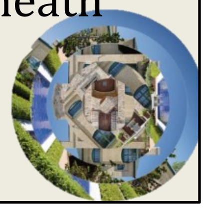
- How does the spin cycle in a washing machine work?

- The drum's normal forces makes the clothes to travel in a circle. The water can go through the holes, so it goes in a straight line. The water is not spun out, the clothes are moved away from the water.

3-02 Centripetal Force

F = ma

Remember the good old days when cars were big, the seats were vinyl bench seats, and there were no seat belts? Well when a guy would take a girl out on a date and he wanted to get cozy, he would put his arm on the back of the seat then make a right hand turn. The car and the guy would turn since the tires and steering wheel provided the centripetal force. The friction between the seat and the girl was not enough, so the girl would continue in a straight path while the car turned underneath her. She would end up in the guy's arms.



After this lesson you will…

- Describe uniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.
- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.

3-03 KINEMATICS OF ROTATIONAL MOTION

OpenStax High School Physics 6.3
OpenStax College Physics 2e 9.1-9.4

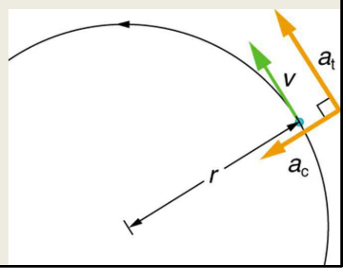
3-03 Kinematics of Rotational Motion

- Rotational motion
- Describes spinning motion
- θ is like x
  - $x = r\theta$ → position
- ω is like v
  - $\omega = \frac{\Delta\theta}{\Delta t}$
  - $v = r\omega$ → velocity
- α is like a
  - $\alpha = \frac{\Delta\omega}{\Delta t}$
  - $a_t = r\alpha$ → acceleration

CCW is +
CW is -

3-03 Kinematics of Rotational Motion

- Two components to acceleration
  - Centripetal
    - Toward center
    - Changes direction only since perpendicular to v
    - $a_c = \frac{v^2}{r}$
- Tangental (linear)
  - Tangent to circle
  - Changes speed only since parallel to v
  - $a_t = r\alpha$



3-03 Kinematics of Rotational Motion

- Equations of kinematics for rotational motion are same as for linear motion

- $\theta = \overline{\omega} t$
- $\omega = at + \omega_0$
- $\theta = \frac{1}{2}\alpha t^2 + \omega_0 t$
- $\omega^2 = \omega_0^2 + 2\alpha\theta$

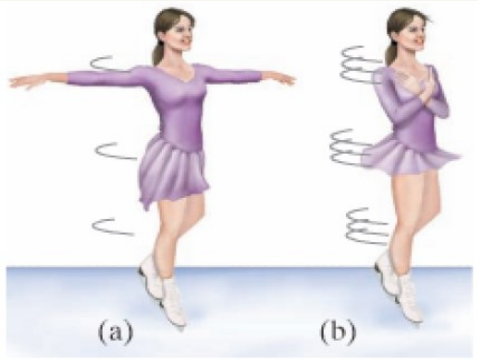
3-03 Kinematics of Rotational Motion

- Reasoning Strategy

1. Examine the situation to determine if rotational motion involved
2. Identify the unknowns (a drawing can be useful)
3. Identify the knowns
4. Pick the appropriate equation based on the knowns/unknowns
5. Substitute the values into the equation and solve
6. Check to see if your answer is reasonable

3-03 Kinematics of Rotational Motion

- A figure skater is spinning at 0.5 rev/s and then pulls her arms in and increases her speed to 10 rev/s in 1.5 s. What was her angular acceleration?
- 39.8 rad/s^2



$$\omega_0 = 0.5\frac{rev}{s}\left(\frac{2\pi\, rad}{rev}\right) = \pi\frac{rad}{s}$$

$$\omega = 10\frac{rev}{s}\left(\frac{(2\pi\, rad)}{rev}\right) = 20\pi\frac{rad}{s}$$

$$t = 1.5\ s$$

$$\omega = \omega_0 + \alpha t$$

$$20\pi\frac{rad}{s} = \pi\frac{rad}{s} + \alpha(1.5\ s)$$

$$19\pi\frac{rad}{s} = \alpha(1.5\ s)$$

$$\alpha = 39.8\frac{rad}{s^2}$$

3-03 Kinematics of Rotational Motion

- A ceiling fan has 4 evenly spaced blades of negligible width. As you are putting on your shirt, you raise your hand. It brushes a blade and then is hit by the next blade. If the blades were rotating at 4 rev/s and stops in 0.01 s as it hits your hand, what angular displacement did the fan move after it hit your hand?
- $\theta = 0.02\ \text{rev} = 0.126\ \text{rad} = 7.2°$

$$\omega_0 = 4\frac{rev}{s}, t = 0.01\ s$$
$$\theta = \bar{\omega} t$$
$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t$$
$$\theta = \left(\frac{0 + 4\frac{rev}{s}}{2}\right)(0.01\ s) = 0.02\ rev = 0.126\ rad = 7.2°$$

$\tau = rF \sin\theta$

After this lesson you will…

- Calculate torque
- Apply torque to equilibrium problems

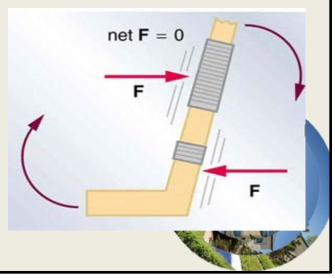
3-04 TORQUE

OpenStax High School Physics 6.3
OpenStax College Physics 2e 10.1-10.2

3-04 Torque

$\tau = rF \sin\theta$

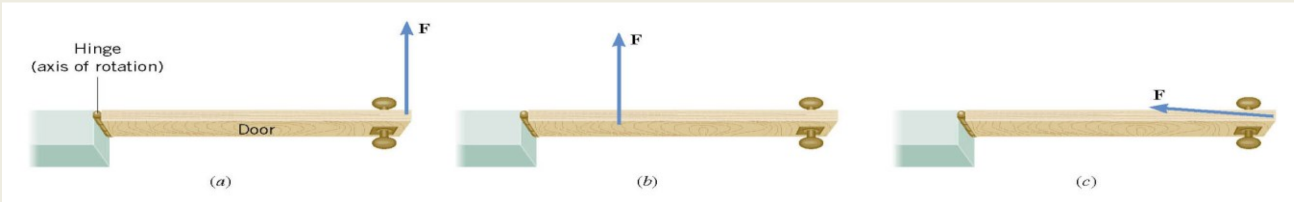
- Statics
  - Study of forces in equilibrium
  - Equilibrium means no acceleration
- First condition of equilibrium
  - $net\ F = 0$
    - $F_x = 0$ and $F_y = 0$
    - They can still rotate, so…



3-04 Torque

$\tau = rF \sin\theta$

- Think of opening a door



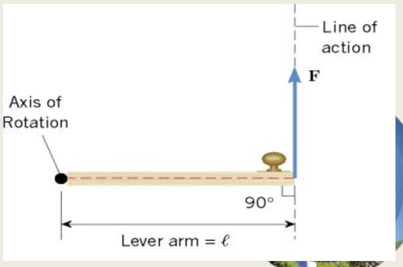
- Which opens the door the best?
  - Picture a
- Big force → large torque
- Force away from pivot → large torque
- Force directed ⊥ to door → large torque

3-04 Torque

$\tau = rF \sin \theta$

- $\tau = r \times F$
  - This means we use the component of the force that is perpendicular to the lever arm
- $\tau = r F_{\perp}$
- $\tau = r F \sin \theta$
  - θ is the angle between the force and the radius

- Unit: Nm
  - CCW → +
  - CW → –



3-04 Torque

$\tau = rF \sin\theta$

- You are meeting the parents of your new "special" friend for the first time. After being at their house for a couple of hours, you walk out to discover the little brother has let all the air out of one of your tires. Not knowing the reason for the flat tire, you decide to change it. You have a 50-cm long lug-wrench attached to a lugnut as shown. If 900 Nm of torque is needed, how much force is needed?
- F = 2078 N
- Less force required if pushed at 90°

$$\tau = rF\sin\theta$$
$$900\ Nm = (0.5m)F\ (\sin 120°)$$
$$F = 2078\ N$$

3-04 Torque

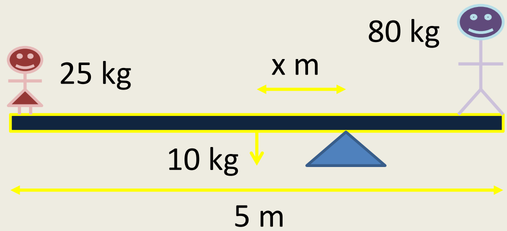
$\tau = rF \sin \theta$

- Second condition of equilibrium
  - Net torque = 0

3-04 Torque

$\tau = rF \sin\theta$

- A 5 m, 10 kg seesaw is balanced by a little girl (25 kg) and her father (80 kg) at opposite ends as shown below. How far from the seesaw's center of mass must the fulcrum be placed?
  - 1.20 m



- How much force must the fulcrum support?
  - 1130 N

$$\sum\tau = 0$$

$$(2.5\,m + x)(25\,kg \cdot 9.8\,m/s^2) + x(10\,kg \cdot 9.8\,m/s^2) - (2.5\,m - x)(80\,kg \cdot 9.8\,m/s^2) = 0$$

$$612.5\,Nm + 245\,N\,x + 98\,N\,x - 1960\,Nm + 784\,N\,x = 0$$

$$-1347.5\,Nm + 1127\,N\,x = 0$$

$$1127\,N\,x = 1347.5\,Nm$$

$$x = 1.20\,m$$

$$\sum F = -W_g - W_f - W_b + F_f = 0$$

$$-(25\,kg)\left(9.8\frac{m}{s^2}\right) - (80\,kg)\left(9.8\frac{m}{s^2}\right) - (10\,kg)\left(9.8\frac{m}{s^2}\right) + F_f = 0$$

$$-1127\,N + F_f = 0$$

$$F_f = 1127\,N$$

$\tau = rF \sin \theta$

After this lesson you will…

- Understand how the moment of inertia affects angular acceleration
- Apply Newton's Second Law for torques ($\tau = I\,\alpha$)

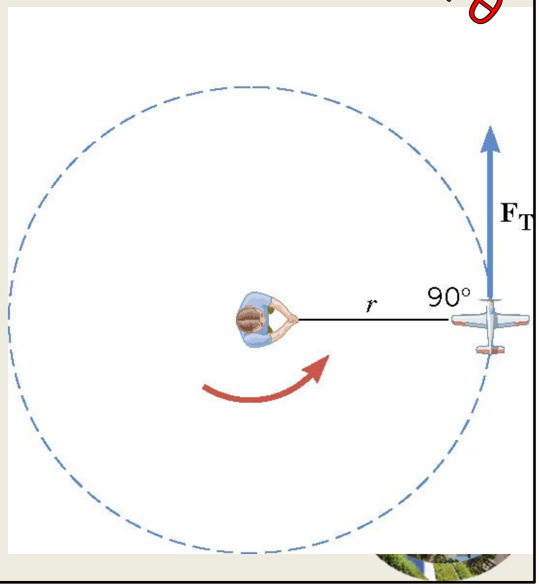
3-05 MOMENT OF INERTIA

Not in OpenStax High School Physics
OpenStax College Physics 2e 10.3

3-05 Moment of Inertia

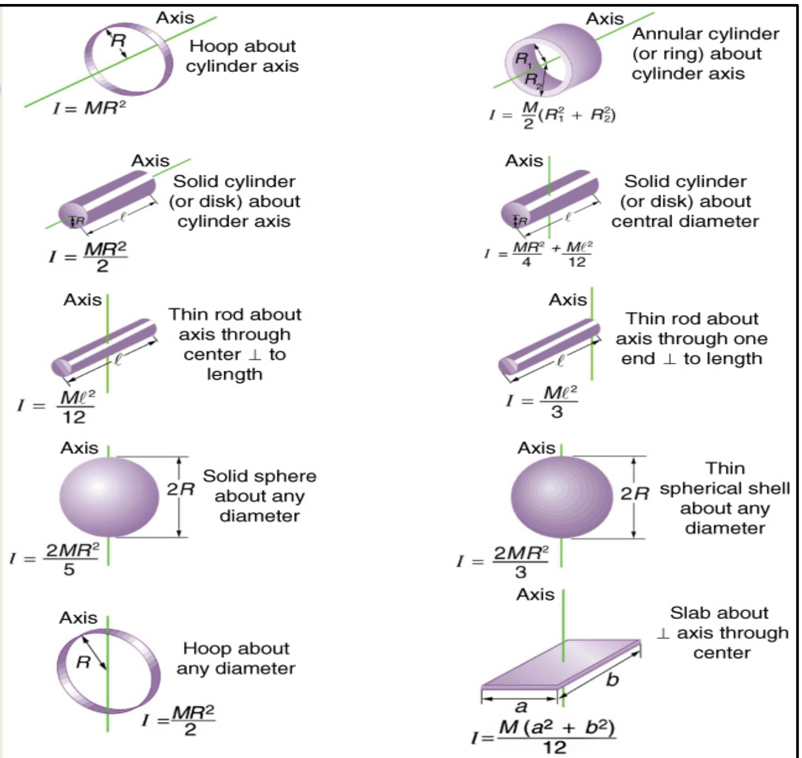
$\tau = rF \sin \theta$

- $\tau = rF_T$
  - $F_T = ma_t$
- $\tau = rma_t$
  - $a_t = r\alpha$
- $\tau = mr^2\alpha$
  - $I = mr^2$ → Moment of inertia of a particle
- $\tau = I\alpha$
  - Newton's second law for rotation
  - α is in rad/s^2



$\tau = rF \sin\theta$ 3-05 Mo

- Moment of Inertia (I) measures how much an object wants to keep rotating (or not start rotating)
- Use calculus to find $I = \sum mr^2$
- Unit:
  - kg m²



5 Moment of Inertia

$\tau = rF \sin \theta$

- The St. Joseph River Swing Bridge in St. Joseph, Michigan has a mass of 300 tons (2.72×10^5 kg) and is 231 ft (70.4 m) long. If the motor produces 563 kNm of torque and takes 10 s to accelerate the bridge to 0.05 rad/s, what is the bridge's moment of inertia?

Due to its well-balanced construction, the 231-foot, 300-ton bridge can be turned with a single 10-horsepower electric motor. It takes approximately 42 seconds to open.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \frac{0.05\ \text{rad/s} - 0\ \text{rad/s}}{10\ \text{s}} = 0.005\frac{rad}{s^2}$$

$$\tau = I\alpha$$

$$563 \times 10^3\ Nm = I\left(0.005\frac{rad}{s^2}\right)$$

$$1.13 \times 10^8\ kg \cdot m^2 = I$$

3-05 Moment of Inertia

$\tau = rF \sin\theta$

- A spinning ride at a carnival is accelerating at 4 rad/s^2. If the ride is shaped like a hoop, and the motor is exerting 128000 Nm of torque, what is the radius of the 500 kg ride?

$$\tau = I\alpha; I = MR^2$$
$$128000\ Nm = (500\ kg)R^2\left(4\frac{rad}{s^2}\right)$$
$$128000\ Nm = 2000\frac{kg}{s^2}R^2$$
$$64\ m^2 = R^2$$
$$8\ m = R$$